MEAN TEMPERATURE DIFFERENCE OF MULTICHANNEL HEAT EXCHANGERS

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The dependence is considered of mean temperature difference on the initial and final temperatures of flow in a heat exchanger with odd number of pipe channels.

The mean temperature difference constitutes one of the main characteristics of a heat exchanger. The relations which link the initial and final temperatures of heat-exchange flows to the mean temperature difference are definitely important in practice. However, one can only compute the mean temperature difference for a specified class of heat exchangers [1-9].

In the present article heat transfer is considered in heat exchangers with one channel releasing the heat and n flow channels taking it in.

The following assumptions are made in solving the problem: 1) the flows and the temperature state are steady; 2) each flow mixes ideally; 3) there is no change in the aggregation state; 4) the specific heat of both flows and the heat-transfer coefficient remain constant; 5) heat exchange surface for all pipe channels is the same; 6) the heat losses are negligibly small.

The heat-balance equation for the entire heat exchanger is of the form

$$C_1(t_{1н} - t_{1к}) = C_2(t_{2к} - t_{2н}) = nF_l lk\overline{\Delta}t. \tag{1}$$

The heat-balance for the right side of the heat exchanger can be written as follows:

$$C_1(t_{1н} - t_1(x)) = C_2 \sum_{i=1}^{n} (-1)^i t_{2i}(x). \tag{2}$$

The heat-balance equations for all elementary length dx are

$$(-1)^{i-1} C_2 dt_{2i}(x) = kF_l (t_1(x) - t_{2i}(x))\, dx, \quad 1 \leqslant i \leqslant n. \tag{3}$$

For a heat exchanger with an even (odd) number of pipe channels one has n = 2N (n = 2N − 1) where N = 1, 2, 3, ...

The case of an odd number of pipe channels is considered. By differentiating (2) and using (3) the following relation is obtained:

$$\frac{dt_1(x)}{dx} = \alpha R \left[(2N-1)\, t_1(x) - \sum_{i=1}^{2N-1} t_{2i}(x)\right], \tag{4}$$

where

$$\alpha = \frac{kF_l}{C_2}.$$

Differentiating now (4) and employing (2) and (3) one arrives at the following second-order differential equation:

$$\frac{d^2 t_1(x)}{dx^2} - (2N-1)\alpha R \frac{dt_1(x)}{dx} + \alpha^2 (R-1)\, t_1(x) = \alpha^2 (R t_{2к} - t_{1н}), \tag{5}$$

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which must be solved under the initial conditions

$$t_1(x)|_{x=0} = t_{1\text{к}}, \quad t_1(x)|_{x=l} = t_{1\text{н}}. \tag{5a}$$

The solution of Eq. (5) together with (5a) is

$$t_1(x) = M_1 e^{\gamma_1 x} + M_2 e^{\gamma_2 x} + \frac{Rt_{2\text{к}} - t_{1\text{н}}}{R-1}, \tag{6}$$

where

$$\gamma_{1,2} = \frac{(2N-1)\alpha}{2}(R \pm \beta), \quad \beta = \sqrt{R^2 - \frac{4(R-1)}{(2N-1)^2}},$$

$$M_1 = \frac{R(t_{2\text{к}} - t_{2\text{н}})}{P(R-1)(e^{\gamma_1 l} - e^{\gamma_2 l})}[1 - P - (1 - PR)e^{\gamma_2 l}],$$

$$M_2 = -\frac{R(t_{2\text{к}} - t_{2\text{н}})}{P(R-1)(e^{\gamma_1 l} - e^{\gamma_2 l})}[1 - P - (1 - PR)e^{\gamma_1 l}].$$

By inserting (6) into (2) and differentiating the expression thus obtained one finds at the point $x = l$

$$M_1\gamma_1 e^{\gamma_1 l} + M_2\gamma_2 e^{\gamma_2 l} = \alpha R\left[(2N-1)t_{1\text{н}} - t_{2\text{к}} - 2\sum_{k=1}^{N-1} t_{\text{к}}\right]. \tag{7}$$

By using Eqs. (3) it is not difficult to evaluate the sum of temperatures at the node points of the right side of the heat exchanger. Then after the substitution of the temperature sum the expression (7) becomes

$$e^{(\gamma_1-\gamma_2)l}\left\{1 - P + \mu_n(1 - PR) - \frac{R-\beta}{2(R-1)}(1 - PR)(e^{\gamma_2 l} + \mu_n) - \frac{R+\beta}{2(R-1)}[1 - P - (1 - PR)e^{\gamma_2 l}]\right\} = 1 - P + \mu_n(1 - PR) - \frac{R-\beta}{2(R-1)}(1 - P)(1 + \mu_n e^{-\gamma_2 l}) + \frac{R+\beta}{2(R-1)}[(1 - P)e^{-\gamma_2 l} - (1 - PR)]\mu_n, \tag{8}$$

where

$$\mu_n = e^{-\frac{1}{(2N-1)\tau}}\left[\frac{1}{N + (N-1)\operatorname{cth}\dfrac{N-1}{(2N-1)\tau} - \operatorname{cth}\dfrac{1}{(2N-1)\tau}} - 1\right],$$

$$\tau = \frac{\overline{\Delta t}}{t_{2\text{к}} - t_{2\text{н}}}.$$

With the aid of heat-balance equation (1) and applying (8) one obtains a transcendental equation for the mean temperature difference

$$\overline{\Delta t} = \{(t_{2\text{к}} - t_{2\text{н}})\beta\} \Big/ \ln\left\{\left[1 - P + \mu_n(1 - PR) - \frac{R-\beta}{2(R-1)}(1 - P)(1 + \mu_n e^{-\gamma_2 l}) + \frac{R+\beta}{2(R-1)} \times [(1 - P)e^{-\gamma_2 l} - (1 - PR)]\mu_n\right] \Big/ \left[1 - P + \mu_n(1 - PR) - \frac{R-\beta}{2(R-1)} \times (1 - PR)(e^{\gamma_2 l} + \mu_n) - \frac{R+\beta}{2(R-1)}[1 - P - (1 - PR)e^{\gamma_2 l}]\right]\right\}. \tag{9}$$

In the particular case of R = 1, Eq. (9) simplifies to

$$\Delta t = \frac{(t_{2\text{к}} - t_{2\text{н}})}{\ln\dfrac{(1-P)(1+\mu_n)\tau[(2N-1)^2 - 1] + \mu_n[P\tau(2N-1)^2 - (1-P)]}{(1-P)(1+\mu_n)\tau[(2N-1)^2 - 1] - [P\tau(2N-1)^2 - (1-P)]}}. \tag{10}$$

It is noted that for heat exchangers with an odd number of pipe channels a change in the direction of motion of the heat delivering flow reduces the mean temperature difference since in the latter case the magnitude of the counter flow is smaller than in the considered case.

By using similar calculations the following formula can be found for the mean temperature difference for a heat exchanger with an even number of pipe channels

$$\overline{\Delta t} = \frac{(t_{2\text{к}} - t_{2\text{н}})\sqrt{R^2 + \frac{1}{N^2}}}{\ln \frac{2 - P\left(1 + R - \sqrt{R^2 + \frac{1}{N^2}} + Q\right)}{2 - P\left(1 + R + \sqrt{R^2 + \frac{1}{N^2}} + Q\right)}}, \tag{11}$$

where

$$Q = \operatorname{cth} \frac{t_{2\text{к}} - t_{2\text{н}}}{2\overline{\Delta t}} - \frac{1}{N} \operatorname{cth} \frac{t_{2\text{к}} - t_{2\text{н}}}{2N\overline{\Delta t}}.$$

The formula (11) is identical with a solution obtained in [8] (under the assumptions 4) and 5)).

It was shown in [3, 7] that a heat exchanger with an infinite number of pipe channels is equivalent to that with crossed fluxes, each heat exchanger mixing at any section. To determine the mean temperature difference in the latter case one has to proceed to the limit $N \to \infty$ in the formulas (9) or (11); this results in

$$\overline{\Delta t} = \frac{(t_{2\text{к}} - t_{2\text{н}})R}{\ln \frac{2 - P\left(1 + \operatorname{cth} \frac{t_{2\text{к}} - t_{2\text{н}}}{2\overline{\Delta t}} - 2\overline{\Delta t}\right)}{2 - P\left(1 + 2R + \operatorname{cth} \frac{t_{2\text{к}} - t_{2\text{н}}}{2\overline{\Delta t}} - 2\overline{\Delta t}\right)}}. \tag{12}$$

To find the mean temperature difference one has to solve the transcendental equations (9)-(12); the latter can be done to the required accuracy on a digital computer.

In practice one evaluates the mean temperature difference by using a correction factor ε which depends only on the dimensionless variables P and R [7, 9],

$$\varepsilon = \frac{\overline{\Delta t}}{(t_{2\text{к}} - t_{2\text{н}})} \cdot \frac{\ln \frac{1 - PR}{1 - P}}{(1 - R)} \tag{13}$$

for $R \neq 1$, but for $R = 1$ one has

$$\varepsilon = \frac{\overline{\Delta t}}{(t_{2\text{к}} - t_{2\text{н}})} \cdot \frac{P}{(1 - P)}. \tag{14}$$

In the diagram (a, b, c, d) are graphs of ε vs P for fixed values of R computed by using (9), (10), (13), (14) for a heat exchanger with 3, 5, 7 and 9 tube channels.

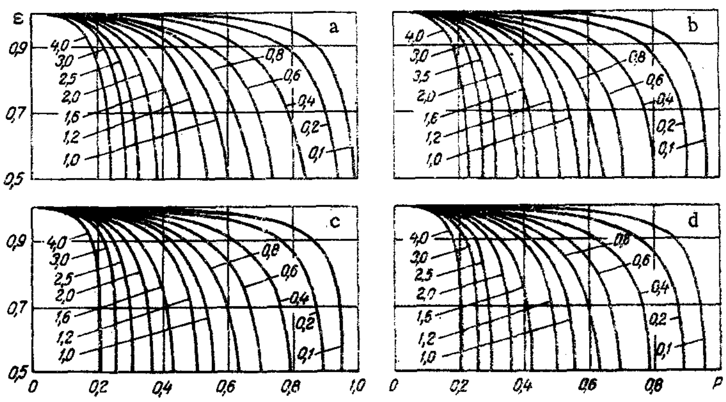

Fig. 1. Correction factor for a heat exchanger with one pipe in the housing case and a) three pipe channels, b) five pipe channels, c) seven pipe channels, d) nine pipe channels. Figures referring to curves give values of R.

NOTATION

n	is the number of channels;
C_1	is the mass velocity of the heat releasing medium;
C_2	is the mass velocity of the heat receiving medium;
t_{1H}, t_{1K}	is the initial and final temperature of the heat releasing medium;
t_{2H}, t_{2K}	is the initial and final temperature of the heat receiving medium;
F_l	is the surface per unit length of channel;
l	is the length of the heat exchanger;
k	is the total heat-transfer coefficient;
$\Delta\bar{t}$	is the mean temperature difference;
$t_{2i}(x)$	is the current temperature in the i-th channel;
t_k	is the temperature at the turning point;
$P = t_{2K} - t_{2H}/t_{1H} - t_{2H}$, $R = t_{1H} - t_{1K}/t_{2K} - t_{2H}$	are the dimensionless variables;
ε	is the correction coefficient.

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